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# Lessons from CLEO and FOCUS Measurements of $D^0 - \overline{D}^0$ Mixing Parameters

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If the true values of the  $D^0 - \overline{D}^0$  mixing parameters lie within the one sigma ranges of recent measurements, then there is strong evidence for a large width difference,  $y \gtrsim 0.01$ , and large  $SU(3)$  breaking effects in strong phases,  $\delta \gtrsim \pi/4$ . These constraints are model independent, and would become stronger if  $|M_{12}/\Gamma_{12}| \ll 1$  in the  $D^0 - \overline{D}^0$  system. The interesting fact that the FOCUS result cannot be explained by a large mass difference is not trivial and depends on the small  $D^0/\overline{D}^0$  production asymmetry in FOCUS and the bounds on CP violating effects from CLEO. The large value of  $\delta$  might help explain why  $y \sim \sin^2 \theta_c$ .

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## 1. Introduction

Recent studies of time-dependent decay rates of  $D^0 \rightarrow K^+\pi^-$  by the CLEO collaboration [1] and measurements of the combination of  $D^0 \rightarrow K^+K^-$  and  $D^0 \rightarrow K^-\pi^+$  rates by the FOCUS collaboration [2] have provided highly interesting results concerning  $D^0 - \overline{D}^0$  mixing. (For previous, related results, see [3-8].) Each of the two experiments finds a signal for mixing at a level that is close to  $2\sigma$ . It is not unlikely that these signals are just the results of statistical fluctuations and the true mixing parameters lie well below the experimental sensitivity. In this work, however, we interpret the experimental results assuming that their central values are not far from the true values and that  $D^0 - \overline{D}^0$  mixing has indeed been observed.

The interpretation of the results and, in particular, testing the consistency of the two recent measurements with each other, require a careful treatment of signs and phase conventions. We present the relevant model-independent formalism in section 2. In section 3 we carefully explain what parameters have the FOCUS and CLEO experiments actually measured. We emphasize that, in principle, both CLEO and FOCUS results can be accounted for even if the width difference is negligibly small. This fact was known for the CLEO result [9], but it is much more subtle for the FOCUS result.

In section 4, we analyze the theoretical implications of the FOCUS and CLEO results in a model independent framework. We do however make some reasonable assumptions. With new physics, it is possible that there are large, CP violating new contributions to the mass difference. On the other hand, it is very unlikely that the width difference [10] and relevant decay amplitudes [11] are significantly affected by new physics. In such a framework, the measured observables depend on the mass difference  $x$ , the width difference  $y$ , two independent CP violating parameters,  $\phi$  and  $A_m$ , and a strong phase  $\delta$ . We find that the experimental results have strong implications for the width difference  $y$  and for the strong phase  $\delta$ . The qualitative features are independent of the other parameters, though the detailed quantitative results are not.

It could be that the  $D^0 - \overline{D}^0$  system is a unique example of a case where the dispersive part of the  $D^0 \rightarrow \overline{D}^0$  transition amplitude is much smaller than the absorptive part,  $|M_{12}| \ll |\Gamma_{12}|$ . (For the  $K^0 - \overline{K}^0$  the two are comparable, while for the  $B^0 - \overline{B}^0$  and  $B_s - \overline{B}_s$

systems the situation is opposite,  $|M_{12}| \gg |\Gamma_{12}|$ .) This situation, which is rarely discussed in the literature, is analyzed in section 5. We point out that, if this approximation is valid, the dependence on  $x$  and on the CP violating parameters can be neglected. Consequently, the FOCUS and CLEO results depend on  $y$  and  $\delta$  only, and the implications become much clearer, both qualitatively and quantitatively.

Within the Standard Model,  $D^0 - \overline{D}^0$  mixing vanishes in the limit of exact  $SU(3)$  flavor symmetry of the strong interactions. For example, the sum of the contributions to the width difference from intermediate  $K^+K^-$ ,  $\pi^+\pi^-$ ,  $K^+\pi^-$  and  $K^-\pi^+$  states vanishes in the  $SU(3)$  limit. The fact that the one sigma ranges of the FOCUS and CLEO results constrain  $\cos\delta$  allows, for the first time, a calculation of this contribution based entirely on experimental data. We carry out such a calculation in section 6 and find a surprisingly large contribution to  $y$ , of order one percent.

A summary of our results is given in section 7.

## 2. Notations and Formalism

We investigate neutral  $D$  decays. The two mass eigenstates,  $|D_1\rangle$  of mass  $m_1$  and width  $\Gamma_1$  and  $|D_2\rangle$  of mass  $m_2$  and width  $\Gamma_2$ , are linear combinations of the interaction eigenstates:

$$\begin{aligned} |D_1\rangle &= p|D^0\rangle + q|\overline{D}^0\rangle, \\ |D_2\rangle &= p|D^0\rangle - q|\overline{D}^0\rangle. \end{aligned} \tag{2.1}$$

The average mass and width are given by

$$m \equiv \frac{m_1 + m_2}{2}, \quad \Gamma \equiv \frac{\Gamma_1 + \Gamma_2}{2}. \tag{2.2}$$

The mass and width difference are parametrized by

$$x \equiv \frac{m_2 - m_1}{\Gamma}, \quad y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma}. \tag{2.3}$$

Decay amplitudes into a final state  $f$  are defined by

$$A_f \equiv \langle f | \mathcal{H}_d | D^0 \rangle, \quad \bar{A}_f \equiv \langle f | \mathcal{H}_d | \overline{D}^0 \rangle. \tag{2.4}$$

It is useful to define the complex parameter  $\lambda_f$ :

$$\lambda_f \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f}. \quad (2.5)$$

The processes that are relevant to the CLEO and FOCUS experiments are the doubly-Cabibbo-suppressed  $D^0 \rightarrow K^+\pi^-$  decay, the singly-Cabibbo-suppressed  $D^0 \rightarrow K^+K^-$  decay, the Cabibbo-favored  $D^0 \rightarrow K^-\pi^+$  decay, and the three CP-conjugate decay processes. We now write down approximate expressions for the time-dependent decay rates that are valid for times  $t \lesssim 1/\Gamma$ . We take into account the experimental information that  $x$ ,  $y$  and  $\tan \theta_c$  are small, and expand each of the rates only to the order that is relevant to the CLEO and FOCUS measurements:

$$\begin{aligned} \Gamma[D^0(t) \rightarrow K^+\pi^-] &= e^{-\Gamma t} |\bar{A}_{K^+\pi^-}|^2 |q/p|^2 \\ &\times \left\{ |\lambda_{K^+\pi^-}^{-1}|^2 + [\mathcal{R}e(\lambda_{K^+\pi^-}^{-1})y + \mathcal{I}m(\lambda_{K^+\pi^-}^{-1})x]\Gamma t + \frac{1}{4}(y^2 + x^2)(\Gamma t)^2 \right\}, \\ \Gamma[\bar{D}^0(t) \rightarrow K^-\pi^+] &= e^{-\Gamma t} |A_{K^-\pi^+}|^2 |p/q|^2 \\ &\times \left\{ |\lambda_{K^-\pi^+}|^2 + [\mathcal{R}e(\lambda_{K^-\pi^+})y + \mathcal{I}m(\lambda_{K^-\pi^+})x]\Gamma t + \frac{1}{4}(y^2 + x^2)(\Gamma t)^2 \right\}, \end{aligned} \quad (2.6)$$

$$\begin{aligned} \Gamma[D^0(t) \rightarrow K^+K^-] &= e^{-\Gamma t} |A_{K^+K^-}|^2 \{1 + [\mathcal{R}e(\lambda_{K^+K^-})y - \mathcal{I}m(\lambda_{K^+K^-})x]\Gamma t\}, \\ \Gamma[\bar{D}^0(t) \rightarrow K^+K^-] &= e^{-\Gamma t} |\bar{A}_{K^+K^-}|^2 \{1 + [\mathcal{R}e(\lambda_{K^+K^-}^{-1})y - \mathcal{I}m(\lambda_{K^+K^-}^{-1})x]\Gamma t\}, \end{aligned} \quad (2.7)$$

$$\begin{aligned} \Gamma[D^0(t) \rightarrow K^-\pi^+] &= e^{-\Gamma t} |A_{K^-\pi^+}|^2, \\ \Gamma[\bar{D}^0(t) \rightarrow K^+\pi^-] &= e^{-\Gamma t} |\bar{A}_{K^+\pi^-}|^2. \end{aligned} \quad (2.8)$$

Within the Standard Model, the physics of  $D^0 - \bar{D}^0$  mixing and of the tree level decays is dominated by the first two generations and, consequently, CP violation can be safely neglected. In all ‘reasonable’ extensions of the Standard Model, the six decay modes of eqs. (2.6), (2.7) and (2.8) are still dominated by the Standard Model CP conserving contributions [11]. On the other hand, there could be new short distance, possibly CP violating contributions to the mixing amplitude  $M_{12}$ . Allowing for only such effects of new physics, the picture of CP violation is simplified since there is no direct CP violation. The

effects of indirect CP violation can be parametrized in the following way [12]:

$$\begin{aligned}
|q/p| &= R_m, \\
\lambda_{K^+\pi^-}^{-1} &= \sqrt{R} R_m^{-1} e^{-i(\delta+\phi)}, \\
\lambda_{K^-\pi^+} &= \sqrt{R} R_m e^{-i(\delta-\phi)}, \\
\lambda_{K^+K^-} &= -R_m e^{i\phi}.
\end{aligned} \tag{2.9}$$

Here  $R$  and  $R_m$  are real and positive dimensionless numbers. CP violation in mixing is related to  $R_m \neq 1$  while CP violation in the interference of decays with and without mixing is related to  $\sin \phi \neq 0$ . The choice of phases and signs in (2.9) is consistent with having  $\phi = 0$  in the Standard Model and  $\delta = 0$  in the  $SU(3)$  limit (see below). We further define

$$\begin{aligned}
x' &\equiv x \cos \delta + y \sin \delta, \\
y' &\equiv y \cos \delta - x \sin \delta.
\end{aligned} \tag{2.10}$$

With our assumption that there is no direct CP violation in the processes that we study, and using the parametrizations (2.9) and (2.10), we can rewrite eqs. (2.6)–(2.8) as follows:

$$\begin{aligned}
\Gamma[D^0(t) \rightarrow K^+\pi^-] &= e^{-\Gamma t} |A_{K^-\pi^+}|^2 \\
&\times \left[ R + \sqrt{R} R_m (y' \cos \phi - x' \sin \phi) \Gamma t + \frac{R_m^2}{4} (y^2 + x^2) (\Gamma t)^2 \right], \\
\Gamma[\overline{D}^0(t) \rightarrow K^-\pi^+] &= e^{-\Gamma t} |A_{K^-\pi^+}|^2 \\
&\times \left[ R + \sqrt{R} R_m^{-1} (y' \cos \phi + x' \sin \phi) \Gamma t + \frac{R_m^{-2}}{4} (y^2 + x^2) (\Gamma t)^2 \right]
\end{aligned} \tag{2.11}$$

$$\begin{aligned}
\Gamma[D^0(t) \rightarrow K^+K^-] &= e^{-\Gamma t} |A_{K^+K^-}|^2 [1 - R_m (y \cos \phi - x \sin \phi) \Gamma t], \\
\Gamma[\overline{D}^0(t) \rightarrow K^+K^-] &= e^{-\Gamma t} |A_{K^+K^-}|^2 [1 - R_m^{-1} (y \cos \phi + x \sin \phi) \Gamma t],
\end{aligned} \tag{2.12}$$

$$\Gamma[D^0(t) \rightarrow K^-\pi^+] = \Gamma[\overline{D}^0(t) \rightarrow K^+\pi^-] = e^{-\Gamma t} |A_{K^-\pi^+}|^2. \tag{2.13}$$

### 3. CLEO and FOCUS Measurements

The FOCUS experiment [2] fits the time dependent decay rates of the singly-Cabibbo suppressed (2.12) and the Cabibbo-favored (2.13) modes to pure exponentials. We define

$\hat{\Gamma}$  to be the parameter that is extracted in this way. More explicitly, for a time dependent decay rate with  $\Gamma[D(t) \rightarrow f] \propto e^{-\Gamma t}(1 - z\Gamma t + \dots)$ , where  $|z| \ll 1$ , we have  $\hat{\Gamma}(D \rightarrow f) = \Gamma(1 + z)$ . The above equations imply the following relations:

$$\begin{aligned}\hat{\Gamma}(D^0 \rightarrow K^+ K^-) &= \Gamma [1 + R_m(y \cos \phi - x \sin \phi)], \\ \hat{\Gamma}(\overline{D}^0 \rightarrow K^+ K^-) &= \Gamma [1 + R_m^{-1}(y \cos \phi + x \sin \phi)], \\ \hat{\Gamma}(D^0 \rightarrow K^- \pi^+) &= \hat{\Gamma}(\overline{D}^0 \rightarrow K^+ \pi^-) = \Gamma.\end{aligned}\tag{3.1}$$

Note that deviations of  $\hat{\Gamma}(D \rightarrow K^+ K^-)$  from  $\Gamma$  do not require that  $y \neq 0$ . They can be accounted for by  $x \neq 0$  and  $\sin \phi \neq 0$ , but then they have a different sign in the  $D^0$  and  $\overline{D}^0$  decays. FOCUS combines the two  $D \rightarrow K^+ K^-$  modes. To understand the consequences of such an analysis, one has to consider the relative weight of  $D^0$  and  $\overline{D}^0$  in the sample. Let us define  $A_{\text{prod}}$  as the production asymmetry of  $D^0$  and  $\overline{D}^0$ :

$$A_{\text{prod}} \equiv \frac{N(D^0) - N(\overline{D}^0)}{N(D^0) + N(\overline{D}^0)}.\tag{3.2}$$

Then

$$\begin{aligned}y_{\text{CP}} &\equiv \frac{\hat{\Gamma}(D \rightarrow K^+ K^-)}{\hat{\Gamma}(D^0 \rightarrow K^- \pi^+)} - 1 \\ &= y \cos \phi \left[ \frac{1}{2}(R_m + R_m^{-1}) + \frac{A_{\text{prod}}}{2}(R_m - R_m^{-1}) \right] \\ &\quad - x \sin \phi \left[ \frac{1}{2}(R_m - R_m^{-1}) + \frac{A_{\text{prod}}}{2}(R_m + R_m^{-1}) \right].\end{aligned}\tag{3.3}$$

The one sigma range measured by FOCUS is

$$y_{\text{CP}} = (3.42 \pm 1.57) \times 10^{-2}.\tag{3.4}$$

The interpretation of this measurement simplifies when the following two facts are taken into account:

- (i) The E687 data [13] suggest that  $A_{\text{prod}}$  is small for FOCUS, of order 0.03.
- (ii) The CLEO data [1] suggest that  $R_m$  is not very different from one (see below). Actually, CLEO implicitly assume that this is the case in their analysis by using

$$R_m^{\pm 2} = 1 \pm A_m.\tag{3.5}$$

Evaluating (3.3) to linear order in the small quantities  $A_{\text{prod}}$  and  $A_m$  yields

$$y_{\text{CP}} = y \cos \phi - x \sin \phi \left( \frac{A_m}{2} + A_{\text{prod}} \right). \quad (3.6)$$

The CLEO measurement [1] gives the coefficient of each of the three terms ( $1$ ,  $\Gamma t$  and  $(\Gamma t)^2$ ) in the doubly-Cabibbo suppressed decays (2.11). Such measurements allow a fit to the parameters  $R$ ,  $R_m$ ,  $x' \sin \phi$ ,  $y' \cos \phi$ , and  $x^2 + y^2$ . Fit A of ref. [1] quotes the following one sigma ranges:<sup>3</sup>

$$\begin{aligned} R &= (0.48 \pm 0.13) \times 10^{-2}, \\ y' \cos \phi &= (-2.5^{+1.4}_{-1.6}) \times 10^{-2}, \\ x' &= (0.0 \pm 1.5) \times 10^{-2}, \\ A_m &= 0.23^{+0.63}_{-0.80}. \end{aligned} \quad (3.7)$$

We would like to point out that the interpretation of the FOCUS and CLEO results in terms of  $y$ ,  $x$ ,  $\phi$ ,  $\delta$  and  $A_m$  is almost independent of our assumption that there is no CP violation in decay. To understand this point, let us parametrize CP violation in decay in the following way:

$$\begin{aligned} A_{\text{CP}}(f) &\equiv \frac{\Gamma(D^0 \rightarrow f) - \Gamma(\overline{D^0} \rightarrow \bar{f})}{\Gamma(D^0 \rightarrow f) + \Gamma(\overline{D^0} \rightarrow \bar{f})} \\ &= \frac{1 - |\bar{A}_{\bar{f}}/A_f|^2}{1 + |\bar{A}_{\bar{f}}/A_f|^2}. \end{aligned} \quad (3.8)$$

Experimentally, we have the following constraints on the asymmetries in the Cabibbo-favored [14], singly-Cabibbo-suppressed [15-17] and doubly-Cabibbo-suppressed [1] decays:

$$\begin{aligned} A_{\text{CP}}(K^- \pi^+) &= 0.001 \pm 0.011, \\ A_{\text{CP}}(K^- K^+) &= 0.0004 \pm 0.0234, \\ A_{\text{CP}}(K^+ \pi^-) &= -0.01 \pm 0.17. \end{aligned} \quad (3.9)$$

For FOCUS, eq. (3.6) would be corrected by terms of order  $A_{\text{CP}}(K^- K^+)A_{\text{prod}}$  and  $A_{\text{CP}}(K^- K^+)A_m$ , which are negligible. For CLEO, the results in eq. (3.7) have been

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<sup>3</sup> CLEO quote a range for  $y'$ . It is obvious however that, with our conventions, their range applies to  $y' \text{sign}(\cos \phi)$  or perhaps to  $y' \cos \phi$ . Since the one sigma range is  $|\cos \phi| \gtrsim 0.8$ , the difference between these two possibilities is unimportant for our purposes.

obtained allowing for CP violation in decay. There is however another subtle aspect of direct CP violation where our theoretical assumption does play a role. In the presence of new CP violating contributions to the decay amplitudes, the CP violating phases in  $\lambda_f$  are not necessarily universal. Therefore, the use of a single phase  $\phi$  in eq. (2.9) and consequently in eqs. (3.6) and (3.7) is valid only in the absence of direct CP violation.

#### 4. Theoretical Interpretation

We now assume that the true values of the various mixing parameters are within the one sigma ranges measured by FOCUS and CLEO. That means in particular that we hypothesize that  $D^0 - \overline{D}^0$  mixing is being observed in the FOCUS measurement of  $y_{\text{CP}}$  and in the CLEO measurement of  $y' \cos \phi$ . The combination of these two results is particularly powerful in its theoretical implications.

Let us first focus on the FOCUS result (3.4). We argue that it is very unlikely that this result is accounted for by the second term in (3.6). Even if we take all the relevant parameters to be close to their one sigma upper bounds, say,  $|x| \sim 0.04$  (we use Fig. 3 of ref. [1] to extract this upper bound),  $|\sin \phi| \sim 0.6$ ,  $|A_m/2| \sim 0.4$  and  $A_{\text{prod}} \sim 0.03$ , we get  $y_{\text{CP}} \sim 0.01$ , about a factor of two too small. We can make then the following *model independent* statement: *if the true values of the mixing parameters are within the one sigma ranges of CLEO and FOCUS measurements, then  $y$  is of order of a (few) percent.* Note that this is true even in the presence of CP violation, which does allow a mass difference,  $x \neq 0$ , to mimic a deviation from the average lifetime. Practically, we can take the FOCUS result to be given to a good approximation by

$$y \cos \phi \approx 0.034 \pm 0.016. \quad (4.1)$$

This is a rather surprising result. Most theoretical estimates are well below the one percent level (for a review, see [18]). These estimates have however been recently criticized [19,20]. We will have more to say about this issue in section 6.

Second, we examine the consistency of the FOCUS and CLEO results. The two most significant measurements, that of  $y \cos \phi$  in eq. (4.1) and that of  $y' \cos \phi$  in eq. (3.7) are



consistent if

$$\cos \delta - (x/y) \sin \delta = -0.73 \pm 0.55. \quad (4.2)$$

This requirement allows us to make a second *model independent* statement: *if the true values of the mixing parameters are within the one sigma ranges of CLEO and FOCUS measurements, then the difference in strong phases between the  $D^0 \rightarrow K^+\pi^-$  and  $D^0 \rightarrow K^-\pi^+$  decays is very large.* For  $\delta = 0$  we get  $y'/y = 1$  instead of the range given in eq. (4.2). To satisfy (4.2), we need, for example,

$$\cos \delta \lesssim \begin{cases} +0.65 & |x| \sim |y|, \\ -0.18 & |x| \ll |y|. \end{cases} \quad (4.3)$$

The result in eq. (4.3) is also rather surprising. The strong phase  $\delta$  vanishes in the  $SU(3)$  flavor symmetry limit [21]. None of the models in the literature [9,22,23] finds such a large  $\delta$ . Eq. (4.3) implies a very large  $SU(3)$  breaking effect in the strong phase. For comparison, the experimental value of  $\sqrt{R} \sim 0.07$  in eq. (3.7) is enhanced compared to its  $SU(3)$  value of  $\tan^2 \theta_c \sim 0.051$  by a factor  $\sim 1.4$ . On the other hand, there are other known examples of  $SU(3)$  breaking effects of order one in  $D$  decays,<sup>4</sup> so perhaps we should not be prejudiced against a very large  $\delta$ .

Before concluding this section, we would like to explain the consequences of the CLEO and FOCUS measurements in the context of the Standard Model. Within the Standard Model,  $D^0 - \overline{D}^0$  mixing and  $D^0$  decays into  $K^+K^-$ ,  $\pi^+\pi^-$  and  $\pi^\pm K^\mp$  are described to an excellent approximation by physics of the first two generations. Consequently, the Standard Model makes a clean prediction that any CP violating effects in these processes are negligibly small. We can thus safely set  $\phi = 0$  and  $R_m = 1$ . The statements below hold in any model where CP is a good symmetry in the relevant processes.

It is important to realize that the choice of  $\phi = 0$  is equivalent to choosing  $|D_1\rangle$  ( $|D_2\rangle$ ) to be the CP-odd (CP-even) state,  $|D_-\rangle$  ( $|D_+\rangle$ ). This can be seen from eq. (2.9). It gives  $\lambda_{K^+K^-} = -1$ . We define the CP-odd state as the mass eigenstate that does not decay into  $K^+K^-$ . Indeed, we now have

$$\langle K^+K^- | \mathcal{H} | D_1 \rangle = p A_{K^+K^-} (1 + \lambda_{K^+K^-}) = 0. \quad (4.4)$$

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<sup>4</sup> For example,  $\Gamma(D^0 \rightarrow K^+K^-)/\Gamma(D^0 \rightarrow \pi^+\pi^-) = 2.75 \pm 0.15 \pm 0.16$  experimentally [15], while the ratio is predicted to be one in the  $SU(3)$  limit.

In the CP limit, a non-zero value of  $y_{\text{CP}}$  (see eq. (3.4)) requires unambiguously that the width difference is large:

$$y = \frac{\Gamma_+ - \Gamma_-}{2\Gamma} = (3.42 \pm 1.39 \pm 0.74) \times 10^{-2}. \quad (4.5)$$

The fact that  $y > 0$  is preferred suggests that the CP-even state has a shorter lifetime, that is  $|D_{+,-}\rangle = |D_{S,L}\rangle$  where  $S$  and  $L$  stands for ‘short’ and ‘long’ lifetimes, respectively. This important result holds in the CP limit model independently.

## 5. The Case of $|M_{12}/\Gamma_{12}| \ll 1$

It could be the case that  $SU(3)$  breaking effects are stronger for the absorptive part of the  $D^0 - \overline{D}^0$  transition amplitude,  $\Gamma_{12}$ , than for the dispersive part,  $M_{12}$ . In this section we investigate the implications of the FOCUS and CLEO results in case that indeed

$$|M_{12}/\Gamma_{12}| \ll 1. \quad (5.1)$$

When we neglect small effects of  $\mathcal{O}(|M_{12}/\Gamma_{12}|)$ , several simplifications occur. Define

$$\phi_{12} \equiv \arg(M_{12}/\Gamma_{12}). \quad (5.2)$$

Then, to leading order in  $|M_{12}/\Gamma_{12}|$ , we have:

$$\begin{aligned} x/y &= 2 |M_{12}/\Gamma_{12}| \cos \phi_{12}, \\ A_m &= 4 |M_{12}/\Gamma_{12}| \sin \phi_{12}, \\ \phi &= -2 |M_{12}/\Gamma_{12}|^2 \sin 2\phi_{12}. \end{aligned} \quad (5.3)$$

We learn that in the limit (5.1),  $x$  can be neglected and all CP violating effects can be neglected. This should be contrasted with the case of  $|\Gamma_{12}/M_{12}| \ll 1$ , which holds for the  $B$  and  $B_s$  mesons, where the effects of  $A_m$  can be neglected but those of  $\phi$  are not suppressed. There are two interesting consequences of this difference. First, in the  $B_s$  system, a lifetime difference between CP eigenstates and flavor specific final states (analogous to  $y_{\text{CP}}$  of eq. (3.3)) measures  $\Delta\Gamma(B_s)$  only if there is no new CP violation in the mixing [24]. In the  $D$  system, if (5.1) holds,  $y_{\text{CP}} \approx y$  model independently. Second, *even in the case that new*

*physics dominates  $M_{12}(D)$ , the sensitivity of any physical observable to it is suppressed by  $|M_{12}/\Gamma_{12}|$ .*

Neglecting  $x$ ,  $A_m$  and  $\phi$ , the FOCUS and CLEO results can be written as follows:

$$\begin{aligned} y &= (3.42 \pm 1.57) \times 10^{-2}, \\ y \cos \delta &= (-2.5^{+1.4}_{-1.6}) \times 10^{-2}, \\ y \sin \delta &= (0.0 \pm 1.5) \times 10^{-2}. \end{aligned} \tag{5.4}$$

The FOCUS measurement determines directly  $y$ . The first two equations give

$$\cos \delta = -0.73^{+0.55}_{-0.27}. \tag{5.5}$$

The third equation requires that  $|\sin \delta|$  is not large and consequently narrows the range for  $\delta$  even further,

$$\cos \delta \lesssim -0.5. \tag{5.6}$$

The conclusion of our discussion here is that if the  $D^0 - \overline{D}^0$  system provides a (unique!) example of  $|M_{12}| \ll |\Gamma_{12}|$ , then the FOCUS and CLEO measurements determine  $y$  to be at the few percent level and the strong phase  $\delta$  is well above  $\pi/2$ .

## 6. Implications for the Width Difference

The value of the phase  $\delta$  has important implications for another aspect of our study, that is the width difference. The contributions of the four charged two-body states,

$$n_{2c} = K^+ K^-, \pi^+ \pi^-, K^+ \pi^-, K^- \pi^+, \tag{6.1}$$

to  $\Gamma_{12}$ , the absorptive part of the transition amplitude  $\langle D^0 | \mathcal{H} | \overline{D}^0 \rangle$ , can be written as

$$(\Gamma_{12})_{2c} = \sum_{n_{2c}} A_{n_{2c}}^* \bar{A}_{n_{2c}}, \tag{6.2}$$

which leads to the following contribution to  $y$ :

$$y_{2c} = \text{BR}(D^0 \rightarrow K^- K^+) + \text{BR}(D^0 \rightarrow \pi^- \pi^+) - 2 \cos \delta \sqrt{R} \text{BR}(D^0 \rightarrow K^+ \pi^-). \tag{6.3}$$

There are two points that we would like to extract from eq. (6.3). First, in the  $SU(3)$  limit,  $\text{BR}(D^0 \rightarrow K^- K^+) = \text{BR}(D^0 \rightarrow \pi^- \pi^+) = \sqrt{R} \text{BR}(D^0 \rightarrow K^+ \pi^-)$ . The phase  $\delta$  defined in (2.9) vanishes in the  $SU(3)$  limit which is consistent with the fact that  $y_{2c} = 0$  in this limit. Second, we can use the measured branching ratios for the four decay modes and the value of the phase  $\delta$  as fitted to the CLEO and FOCUS results to estimate  $y_{2c}$ . We use [25]

$$\begin{aligned}\text{BR}(D^0 \rightarrow K^- \pi^+) &= (3.83 \pm 0.09) \times 10^{-2}, \\ \text{BR}(D^0 \rightarrow \pi^- \pi^+) &= (1.52 \pm 0.09) \times 10^{-3}, \\ \text{BR}(D^0 \rightarrow K^- K^+) &= (4.24 \pm 0.16) \times 10^{-3},\end{aligned}\tag{6.4}$$

and [1] (see eq. (3.7))

$$\sqrt{R} = 0.069 \pm 0.009.\tag{6.5}$$

Using central values for the branching ratios, we get:

$$y_{2c} \sim (5.76 - 5.29 \cos \delta) \times 10^{-3}.\tag{6.6}$$

Taking  $-1 \lesssim \cos \delta \lesssim 0$  from (4.3), we find

$$0.6 \times 10^{-2} \lesssim y_{2c} \lesssim 1.1 \times 10^{-2},\tag{6.7}$$

to be compared with the range (4.5) for  $y$ . Note that the sign of this contribution is consistent with the overall sign of  $y$  as measured by FOCUS. There are of course other intermediate states that contribute to  $y$ . Eq. (6.7) suggests that, if the strong phases strongly violate  $SU(3)$  as required for consistency of the CLEO and FOCUS results, such contributions could easily be at the percent level as required by the same experiments.

## 7. Conclusions

The FOCUS and CLEO collaborations have provided new measurements of the  $D^0 - \overline{D}^0$  mixing parameters that are sensitive to effects of order a few percent. FOCUS obtains a  $2.2\sigma$  signal and CLEO obtains a  $1.8\sigma$  signal of such effects. It could well be that these signals are just statistical fluctuations and that the mixing parameters are much smaller

than the percent level. This is the theoretical wisdom, based on the Standard Model and on approximate flavor  $SU(3)$ . If, however, the central values of the two measurements are close to the true values, then at least the assumption of approximate  $SU(3)$  for the strong interactions has to be modified. In particular, there are two *independent* pieces of evidence that the strong phase in  $D \rightarrow K^\pm \pi^\mp$  decays is very large,  $\delta \gtrsim \pi/4$  and perhaps  $\delta \sim 3\pi/4$  (while  $\delta = 0$  in the  $SU(3)$  limit):

- (i) Either a negative sign for  $\cos \delta$  or large  $x$  and large  $\sin \delta$  are necessary to make the signs of the mixing parameters measured by FOCUS and by CLEO consistent with each other.
- (ii)  $\cos \delta$  far from its  $SU(3)$  limit value of one implies that some contributions to the width difference are at the percent level.

We also discussed the possibility that in the  $D^0 - \overline{D}^0$  system  $|M_{12}/\Gamma_{12}| \ll 1$ , in contrast to the neutral  $B$  meson systems. In such a case, the  $D^0 - \overline{D}^0$  system is not sensitive to new physics, even if new physics dominates  $M_{12}$ . In particular, CP is expected to be a good symmetry regardless of whether there are large CP violating contributions to  $M_{12}$ . The above statements about large  $SU(3)$  breaking effects become even sharper in this case.

A much clearer picture would emerge if the accuracy of the measurements improves and, in particular, if the mixing parameters are measured separately in the  $D^0$  and  $\overline{D}^0$  decays. For example, the FOCUS collaboration has summed over the  $D^0 \rightarrow K^+ K^-$  and  $\overline{D}^0 \rightarrow K^+ K^-$  modes, but there is much to learn from comparing them to each other. Explicitly, we obtain from eq. (3.1):

$$\frac{\hat{\Gamma}(D^0 \rightarrow K^+ K^-) - \hat{\Gamma}(\overline{D}^0 \rightarrow K^+ K^-)}{\hat{\Gamma}(D^0 \rightarrow K^+ K^-) + \hat{\Gamma}(\overline{D}^0 \rightarrow K^+ K^-)} = \frac{A_m}{2} y \cos \phi - x \sin \phi. \quad (7.1)$$

A difference between the fitted decay width of the two CP conjugate modes will provide important information on the CP violating parameters.

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